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CYCLIC UNIAXIAL AND BIAXIAL HARDENING OF TYPE 304 STAINLESS STEEL MODELED BY THE VISCOPLASTICITY THEORY BASED ON OVERSTRESS

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The isotropic theory of viscoplasticity based on overstress does not use a yield surface or a loading and unloading criterion. The inelastic strain rate depends on overstress, the difference between the stress and the equilibrium stress, and is assumed to be rate-dependent. Special attention is paid to the modeling of elastic regions.

For the modeling of cyclic hardening, such as observed in annealed Type 304 Stainless Steel, an additional growth law for a scalar quantity which represents the rate-independent asymptotic value of the equilibrium stress is added. It is made to increase with inelastic deformation using a new scalar measure which differentiates between nonproportional and proportional loading.

The theory is applied to correlate uniaxial data under two-step amplitude loading including the effect of further hardening at the high amplitude and proportional and nonproportional cyclic loadings. Results are compared with corresponding experiments.

INTRODUCTION

For the modeling of the rate(time)-dependent, cyclic neutral, inelastic deformation behavior of metals, the theory of viscoplasticity based on overstress (VBO) with a differential growth law for the equilibrium stress was proposed [1]. When compared with biaxial experiments it was shown to predict the room temperature deformation behavior of an Aluminum alloy under both monotonic and cyclic proportional and nonproportional loadings [2].

Some alloys such as annealed copper [3], Type 304 Stainless Steel [4,5] and 316L Stainless Steel [6,7] exhibit complicated cyclic hardening phenomena. For their modeling an additional growth law for a scalar quantity is introduced in VBO. Its growth with inelastic deformation is governed by a new scalar measure which differentiates between proportional and nonproportional loadings. The effect is similar to isotropic hardening in classical plasticity. Unlike other approaches, e.g. [6], neither an updating rule (or help function) nor a loading-unloading criterion is needed in this formulation.

The purposes of the present paper are to give an isotropic formulation of VBO applicable for cyclic hardening and to demonstrate its predictive capability in proportional and nonproportional strain-controlled cyclic loadings. Some results of numerical experiments are compared with corresponding room temperature results on Type 304 Stainless Steel.

The isotropic formulation consists of the following differential equations and functions:

$$\dot{\underset{\sim}{e}} = \frac{1+\nu}{E}\dot{\underset{\sim}{s}} + \frac{1+\nu}{Ek[\Gamma_o]}(\underset{\sim}{s} - \underset{\sim}{g}^d) \quad , \tag{1}$$

$$\dot{\underset{\sim}{g}} = \psi[\Gamma_o, A]\underset{\sim\sim}{T}\dot{\underset{\sim}{\varepsilon}} - (\underset{\sim}{g} - E_t\underset{\sim\sim}{T}\underset{\sim}{\varepsilon})\frac{\dot{\phi}}{b[\Gamma_o, A]} \quad , \tag{2}$$

$$tr(\dot{\underset{\sim}{\varepsilon}}) = \frac{1-2\nu}{E}tr(\dot{\underset{\sim}{\sigma}}) \quad , \tag{3}$$

$$b[\Gamma_o, A] = \frac{A}{\psi[\Gamma_o, A] - E_t} \quad , \tag{4}$$

$$\Gamma_o = \left(\frac{3}{2}tr(\underset{\sim}{s} - \underset{\sim}{g}^d)^2\right)^{1/2} \quad , \tag{5}$$

$$\dot{\phi} = \left(\frac{2}{3}tr(\dot{\underset{\sim}{\varepsilon}}^{in})^2\right)^{1/2} \quad , \tag{6}$$

$$\dot{P}_i = \left(tr\left(\dot{\underset{\sim}{\varepsilon}}^{in}(\underset{\sim}{\varepsilon}^{in})^2\,\dot{\underset{\sim}{\varepsilon}}^{in}\right)\right)^{1/2} \quad , \tag{7}$$

$$\dot{P}_o = \left(tr(\underset{\sim}{\Omega}\underset{\sim}{\Omega}^T)\right)^{1/2} \quad , \tag{8}$$

$$\underset{\sim}{\Omega} = \underset{\sim}{\varepsilon}^{in}\dot{\underset{\sim}{\varepsilon}}^{in} - \dot{\underset{\sim}{\varepsilon}}^{in}\underset{\sim}{\varepsilon}^{in} \quad , \tag{9}$$

$$\dot{A}^* = a_1\dot{P}_i^{\,a_2} + a_5\dot{P}_o^{\,a_6} - a_3(A - A_o)^{a_4} \quad , \tag{10}$$

$$\dot{A} = a_7|\dot{A}^*| + a_8\dot{A}^* \quad , \tag{11}$$

where $T_{ijk\ell} = \frac{1}{1+\nu}\left(\frac{1}{2}(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk}) + \frac{\nu}{1-2\nu}\delta_{ij}\delta_{k\ell}\right)$, $a_7 + a_8 = 1$ and $a_8 - a_7 = u$ $(0 \le u \le 1)$. For cyclic neutral behavior the constants a_1, a_3, a_5 are zero; a_7, a_8, and Eqs.(7)-(11) are not necessary.

The system of differential equations introduced in the above is similar to that obtained in [2] with the exception that a dependence of ψ on A and a growth law for A are added. The former ensures that the linear elastic regions expand when A increases. In addition to the inelastic strain path length defined in (6), two additional scalar measures are introduced in (7) and (8). The last measure is nonzero when the inelastic strain and the inelastic strain rate are not collinear. This occurs in nonproportional loading. The quantity defined in (7) is of the same order as the one defined in (8) and accumulates in every inelastic loading. These quantities have been used to correlate experimental data [8].

To illustrate the capability of the newly proposed growth law for A consider two axial-torsion tests; one with proportional straining and the other with 90° out-of-phase straining. It is clear that, though $\dot{P}_i$ are not zero, $\dot{P}_o = 0$ in the former case and $\dot{P}_o \neq 0$ in the latter case. Therefore Eq.(10) gives a different growth in the two cases which is reflected in the magnitude of $\dot{P}_o$. If we consider two cyclic tests with different strain ranges, again Eq.(10) gives a different growth depending on the inelastic strain because ε^{in} is included in $\dot{P}_o$ and $\dot{P}_i$. The model can predict a further hardening at the high amplitude under two-step amplitude loading even if saturation was reached at the first amplitude. If a small strain range is performed following a large strain range in the two-step amplitude loading, the model predicts a stabilized stress corresponding to the most recent level irrespective of the prior history. Equation (11) is introduced to delay the process of reaching the stabilized value.

Aside from these qualitative predictions, the details of deformation behavior must be evaluated through numerical experiments. The constants and functions of the theory were selected to represent the Type 304 Stainless Steel. All numerical integrations were performed using IMSL routine DGEAR on an IBM AT personal computer.

NUMERICAL EXPERIMENTS AND DISCUSSIONS

The procedure introduced in [1,2] for determining the constants and functions is still useful. Stabilized stresses for different strain ranges under both uniaxial and 90° out-of-phase loadings are necessary for the identifications of the constants associated with the growth law for A. The details can be found in [9]. The constants and functions for annealed Type 304 SS are listed in Table 1.

Simulations of the following four tests are reported, all conducted at the same equivalent strain rate of $\dot{\varepsilon}_e = 0.0003\ s^{-1}$. The first test is a two amplitude step-up uniaxial test with $\varepsilon_a = 0.0056$ for 15 cycles followed by 15 cycles with $\varepsilon_a = 0.008$. In the second test the sequence of the applied strain amplitudes is reversed. In the third test the second block consisted of a 90° out-of-phase loading for 5 cycles with the same equivalent strain amplitude as the previous uniaxial cycling to near saturation. Lastly a 90° out-of-phase cyclic test without any prior deformation was performed with $\varepsilon_e = 0.0056$ for 5 cycles.

The results for the first three tests are presented in Figure 1. The theory correlates the experimental result reasonably well in normal cyclic hardening tests and gives similar responses as reported in [6] in both further hardening and partial fading memory cases. It also demonstrates that an additional hardening is experienced in 90 degrees out-of-phase loading even when the material had almost saturated under proportional loading with the same strain range. This behavior is found in experiments [3,5]. It was shown [5] that the cyclic hardening behavior during in-phase loading (axial, torsional and proportional loading) can be correlated on the basis of the v. Mises equivalent stress and the accumulated strain path length, the integral of (6). The present theory uses these quantities, see (5) and (6). On the basis of the results in Fig. 1 it is reasonable to assume that the theory can correlate the hardening in in-phase loading.

The correlation for the 90° out-of-phase loading is shown in Figure 2 where the experimental result for the 5th cycle is also plotted. Comparison of the saturation levels for out-of-phase loading in Fig. 1 (path AB") and Fig. 2 (saturation is almost reached after 5 cycles) shows that they are almost equal. It was found in [3] that the saturated stress was not dependent on prior history. This fact is represented by the present theory. Even though the model gives a correct stabilized stress in 90° out-of-phase loading, the description of the transient behavior needs improvement.

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TABLE 1 MATERIAL CONSTANTS AND FUNCTIONS*

Material constants:

$E = 195000$ MPa	$a_3 = 0.0653$ MPa$^{1-a_4}$ s^{-1}
$E_t = 2000$ MPa	$a_4 = 0.703$
$A_o = 115$ MPa	$a_5 = 41.17$ MPa s$^{-(1-a_6)}$
$\nu = 0.5$	$a_6 = 0.2062$
$a_1 = 380000$ MPa s$^{-(1-a_2)}$	$a_7 = 0.495$
$a_2 = 0.925$	$a_8 = 0.505$

Viscosity function:

$$k[x] = k_1\left(1+\frac{|x|}{k_2}\right)^{-k_3} ,$$

k_1=314200 s, k_2=60 MPa, k_3=21.98 .

Shape modulus function:

$$\psi[x,y] = c_1[y] + (c_2-c_1[y])\exp(-c_3x) ,$$

$$c_1[y] = H_1 + H_2y ,$$

c_2=182500 MPa, c_3=0.0783 MPa^{-1}, H_1=74740 MPa, H_2=37.04

*All x and y are in units of MPa.

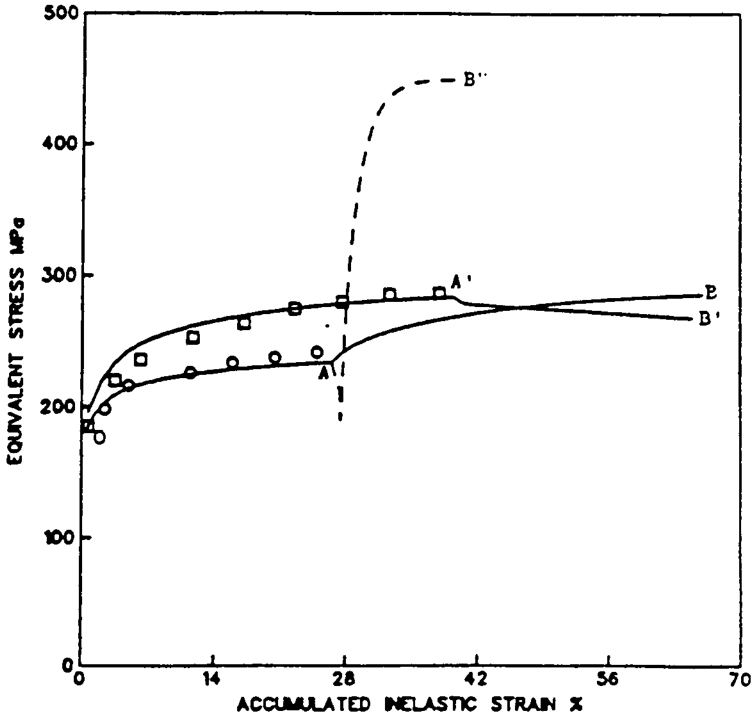

Figure 1. The equivalent stress vs accumulated inelastic strain.
Solid and dotted lines: theory.
Strain history:
OAB uniaxial cycling with amplitude increase
- 15 cycles with $\varepsilon_a = 0.0056$ followed by
15 cycles with $\varepsilon_a = 0.008$;
OA'B' uniaxial cycling with amplitude decrease
- 15 cycles with $\varepsilon_a = 0.008$ followed by
15 cycles with $\varepsilon_a = 0.0056$;
OAB'' uniaxial cycling followed by 90° out-of-phase
cycling for 5 cycles at the same effective
strain amplitude.
All tests were performed with $\dot{\varepsilon}_e = 0.0003\ s^{-1}$.

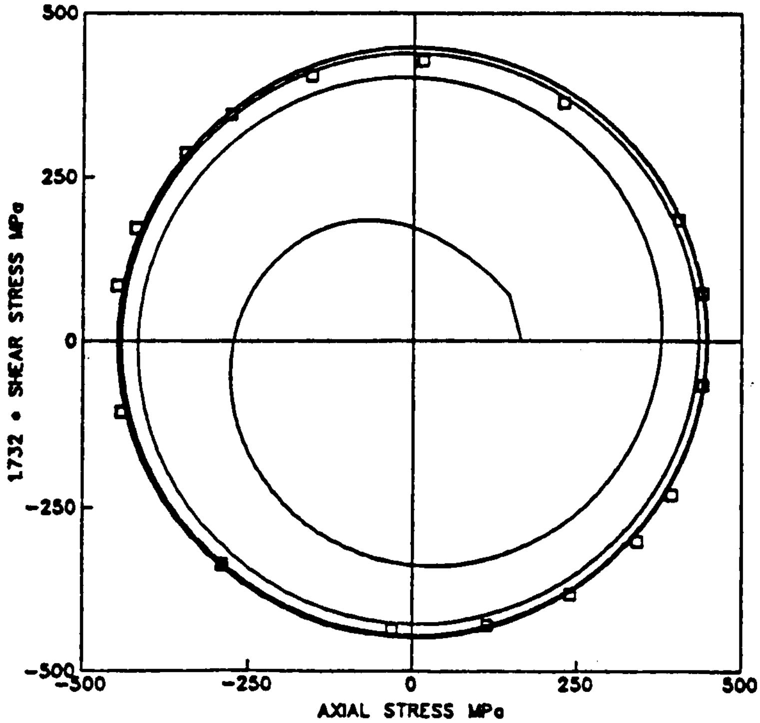

Figure 2. The $\sqrt{3}$ × shear stress vs axial stress under 90° out-of-phase loading with $\varepsilon_e = 0.0056$ and $\dot{\varepsilon}_e = 0.0003\ s^{-1}$. Solid lines: theory. Experimental data only show the 5th cycle.